Solving Systems of Inequalities by Graphing

Main Ideas

- Solve systems of inequalities by graphing.
- Determine the coordinates of the vertices of a region formed by the graph of a system of inequalities.

New Vocabulary

system of inequalities

During one heartbeat, blood pressure reaches a maximum pressure and a minimum pressure, which are measured in millimeters of mercury (mm-Hg). It is expressed as the maximum over the minimum—for example, 120/80. Normal blood pressure for people under 40 ranges from 100 to 140 mm Hg for the maximum and from 60 to 90 mm Hg for the minimum. This can be represented by a system of inequalities.
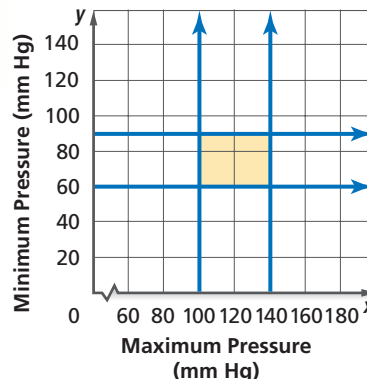
Graph Systems of Inequalities To solve a **system of inequalities**, we need to find the ordered pairs that satisfy all of the inequalities in the system. The solution set is represented by the intersection of the graphs of the inequalities.

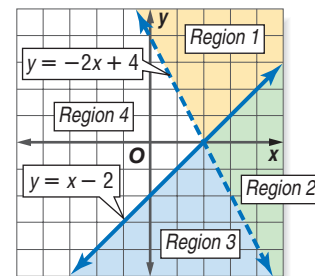### EXAMPLE   Intersecting Regions

1 Solve each system of inequalities.

a. $y > -2x + 4$

$y \leq x - 2$

Solution of $y > -2x + 4 \rightarrow$ Regions 1 and 2

Solution of $y \leq x - 2 \rightarrow$ Regions 2 and 3

The region that provides a solution of both inequalities is the solution of the system. Region 2 is the solution of the system.
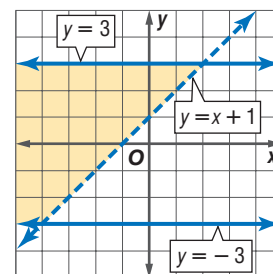

b. $y > x + 1$

$|y| \leq 3$

The inequality $|y| \leq 3$ can be written as $y \leq 3$ and $y \geq -3$.

Graph all of the inequalities on the same coordinate plane and shade the region or regions that are common to all.

Study Tip

Look Back

To review **graphing inequalities**, see Lesson 2-7.

Concepts in Motion

Animation algebra2.com

CHECK Your Progress

1A. $y \leq 3x - 4$

$y > -2x + 3$

1B. $|y| < 3$

$y \geq x - 1$

It is possible that two regions do *not* intersect. In such cases, we say the solution set is the empty set (Ø) and no solution exists.
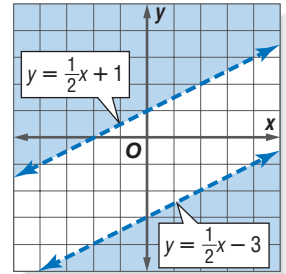
EXAMPLE   **Separate Regions**

2 Solve the system of inequalities by graphing.

$$y > \frac{1}{2}x + 1$$

$$y < \frac{1}{2}x - 3$$

Graph both inequalities. The graphs do not overlap, so the solution sets have no points in common. The solution set of the system is Ø.

CHECK Your Progress

2. $y > \frac{1}{4}x + 4$

  $y < \frac{1}{4}x - 2$

Real-World EXAMPLE   **Write and Use a System of Inequalities**

3 **BASKETBALL** The 2005–06 Denver Nuggets roster included players of varying weights and heights. Francisco Elson was the largest at 7′0″ and 235 pounds. The smallest player on the team was Earl Boykins at 5′5″ and 133 pounds. Write and graph a system of inequalities that represents the range of heights and weights for the members of the team.

Let h represent the height of a member of the Denver Nuggets. The possible heights for a member of the team are at least 65 inches, but no more than 84 inches. We can write two inequalities.

$h \geq 65$ and $h \leq 84$

Let w represent the weights of a player on the Denver Nuggets. The weights can be written as two inequalities.

$w \geq 133$ and $w \leq 235$

Graph all of the inequalities. Any ordered pair in the intersection of the graphs is a solution of the system. In this case, a solution of the system of inequalities is a potential height and weight combination for a member of the Denver Nuggets.
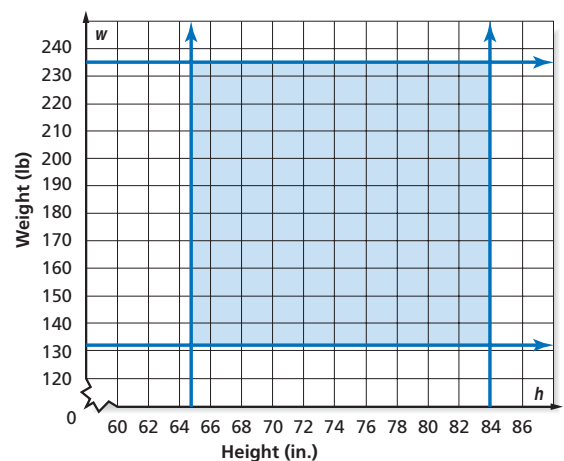
CHECK Your Progress

3. **CATERING** Classy Catering needs at least 15 food servers and 5 bussers to cater a large party. But in order to make a profit, they can have no more than 34 food servers and 7 bussers working at an event. Write and graph a system of inequalities that represents this information.

Find Vertices of a Polygonal Region Sometimes, the graph of a system of inequalities forms a polygonal region. To find the vertices of the region, determine the coordinates of the points at which the boundaries intersect.

EXAMPLE · Find Vertices

4 **GEOMETRY** Find the coordinates of the vertices of the figure formed by $x + y \geq -1$, $x - y \leq 6$, and $12y + x \leq 32$.

Graph each inequality. The intersection of the graphs forms a triangle.

The coordinates $(-4, 3)$ and $(8, 2)$ can be determined from the graph. To find the third vertex, solve the system of equations $x + y = -1$ and $x - y = 6$.

Add the equations to eliminate y.

$$x + y = -1$$
$$\underline{(+)\ x - y = \quad 6}$$
$$2x \quad = \quad 5 \quad \text{Add the equations.}$$
$$x = \frac{5}{2} \quad \text{Divide each side by 2.}$$

Now find y by substituting $\frac{5}{2}$ for x in the first equation.

$$x + y = -1 \quad \text{First equation}$$
$$\frac{5}{2} + y = -1 \quad \text{Replace } x \text{ with } \frac{5}{2}.$$
$$y = -\frac{7}{2} \quad \text{Subtract } \frac{5}{2} \text{ from each side.}$$

CHECK Compare the coordinates to the coordinates on the graph.

The x-coordinate of the third vertex is between 2 and 3, so $\frac{5}{2}$ is reasonable. The y-coordinate of the third vertex is between -3 and -4, so $-\frac{7}{2}$ is reasonable.

The vertices of the triangle are at $(-4, 3)$, $(8, 2)$, and $\left(\frac{5}{2}, -\frac{7}{2}\right)$.
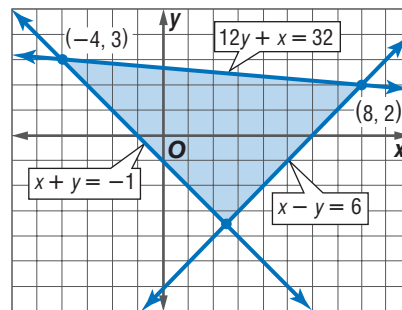
CHECK Your Progress

4. Find the coordinates of the vertices of the figure formed by $x + y \leq 2$, $x - 2y \leq 8$, and $x + \left(-\frac{1}{3}y\right) \geq -\frac{2}{3}$.

Online Personal Tutor at algebra2.com

CHECK Your Understanding

Examples 1, 2
(pp. 130–131)

Solve each system of inequalities by graphing.

1. $x \leq 4$
$y > 2$

2. $y \leq -4x - 3$
$y > -4x + 1$

3. $|x - 1| \leq 2$
$x + y > 2$

4. $y \geq 3x + 3$
$y < 3x - 2$

Example 3
(p. 131)

SHOPPING For Exercises 5 and 6, use the following information.
The most Jack can spend on bagels and muffins for the cross country team is
$28. A package of 6 bagels costs $2.50. A package of muffins costs $3.50 and
contains 8 muffins. He needs to buy at least 12 bagels and 24 muffins.

5. Graph the region that shows how many packages of each item he can
purchase.

6. Give an example of three different purchases he can make.

Example 4
(p. 132)

**Find the coordinates of the vertices of the figure formed by each system
of inequalities.**

7. $y \leq x$

$y \geq -3$

$3y + 5x \leq 16$

8. $y \geq x - 3$

$y \leq x + 7$

$x + y \leq 11$

$x + y \geq -1$

Exercises

Solve each system of inequalities by graphing.

9. $x \geq 2$

$y > 3$

10. $x \leq -1$

$y \geq -4$

11. $y < 2 - x$

$y > x + 4$

12. $x > 1$

$x \leq -1$

13. $3x + 2y \geq 6$

$4x - y \geq 2$

14. $4x - 3y < 7$

$2y - x < -6$

15. $3y \leq 2x - 8$

$y \geq \frac{2}{3}x - 1$

16. $y > x - 3$

$|y| \leq 2$

17. $2x + 5y \leq -15$

$y > \frac{-2}{5}x + 2$

18. PART-TIME JOBS Rondell makes $10 an hour cutting grass and $12 an hour
for raking leaves. He cannot work more than 15 hours per week. Graph
two inequalities that Rondell can use to determine how many hours he
needs to work at each job if he wants to earn at least $120 per week.

19. RECORDING Jane's band wants to spend no more than $575 recording their
first CD. The studio charges at least $35 an hour to record. Graph a system
of inequalities to represent this situation.

**Find the coordinates of the vertices of the figure formed by each system
of inequalities.**

20. $y \geq 0$

$x \geq 0$

$x + 2y \leq 8$

21. $y \geq -4$

$y \leq 2x + 2$

$2x + y \leq 6$

22. $x \leq 3$

$-x + 3y \leq 12$

$4x + 3y \geq 12$

23. $x + y \leq 9$

$x - 2y \leq 12$

$y \leq 2x + 3$

24. GEOMETRY Find the area of the region defined by the system of inequalities
$y + x \leq 3$, $y - x \leq 3$, and $y \geq -1$.

25. GEOMETRY Find the area of the region defined by the system of inequalities
$x \geq -3$, $y + x \leq 8$, and $y - x \geq -2$.

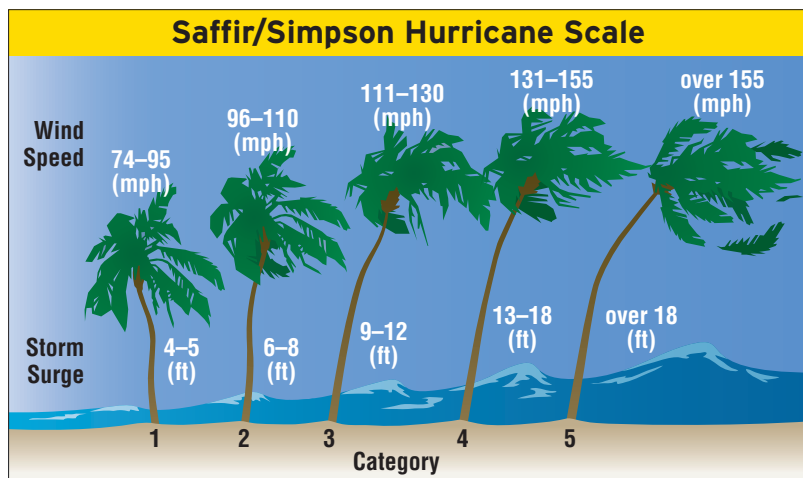**HURRICANES** For Exercises 26 and 27, use the following information.
Hurricanes are divided into five categories according to their wind speed and storm surge. Category 5 is the most destructive type of hurricane.

Saffir/Simpson Hurricane Scale

Source: National Oceanic and Atmospheric Administration

26. Write and graph the system of inequalities that represents the range of wind speeds s and storm surges h for a Category 3 hurricane.

27. On August 29, 2005, Hurricane Katrina hit the Gulf coasts of Louisiana and Mississippi. At its peak, Katrina had maximum sustained winds of 145 mph. Classify the strength of Hurricane Katrina and state the expected heights of its storm surges.

BAKING For Exercises 28–30, use the recipes at the right.
The Merry Bakers are baking pumpkin bread and Swedish soda bread for this week's specials. They have at most 24 cups of flour and at most 26 teaspoons of baking powder on hand.

28. Graph the inequalities that represent how many loaves of each type of bread the bakers can make.

29. List three different combinations of breads they can make.

30. Which combination uses all of the available flour and baking soda?

Pumpkin Bread

2 c. of flour

1 tsp. baking powder

Swedish Soda Bread

$1\frac{1}{2}$ c. of flour

$2\frac{1}{2}$ tsp. baking powder

Solve each system of inequalities by graphing.

31. $y < 2x - 3$
$y \leq \frac{1}{2}x + 1$

32. $|x| \leq 3$
$|y| > 1$

33. $|x + 1| \leq 3$
$x + 3y \geq 6$

34. $y \geq 2x + 1$
$y \leq 2x - 2$
$3x + y \leq 9$

35. $x - 3y > 2$
$2x - y < 4$
$2x + 4y \geq -7$

36. $x \leq 1$
$y < 2x + 1$
$x + 2y \geq -3$

H.O.T. Problems

37. OPEN ENDED Write a system of inequalities that has no solution.

38. REASONING Determine whether the following statement is *true* or *false*. If false, give a counterexample. *A system of two linear inequalities has either no points or infinitely many points in its solution.*

Doug Martin

39. CHALLENGE Find the area of the region defined by $|x| + |y| \leq 5$ and $|x| + |y| \geq 2$.

40. *Writing in Math* Using the information about blood pressure on page 130, explain how you can determine whether your blood pressure is in a normal range utilizing a graph of the system of inequalities.

A STANDARDIZED TEST PRACTICE

41. ACT/SAT Choose the system of inequalities whose solution is represented by the graph.

A $y < -2$
  $x < -3$

B $y \leq -2$
  $x > -3$

C $x \leq -2$
  $y > -3$
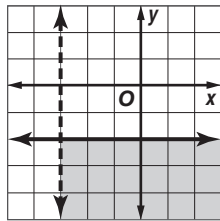
D $x < -3$
  $y < -3$

42. REVIEW To be a member of the marching band, a student must have a GPA of at least 2.0 and must have attended at least five after-school practices. Choose the system of inequalities that best represents this situation.

F $x \geq 2$
  $y \geq 5$

G $x \leq 2$
  $y \leq 5$

H $x < 2$
  $y < 5$

J $x > 2$
  $y > 5$

Spiral Review

Solve each system of equations by using either substitution or elimination. (Lesson 3-2)

43. $4x - y = -20$
  $x + 2y = 13$

44. $3x - 4y = -2$
  $5x + 2y = 40$

45. $4x + 5y = 7$
  $3x - 2y = 34$

Solve each system of equations by graphing. (Lesson 3-1)

46. $y = 2x + 1$
  $y = -\frac{1}{2}x - 4$

47. $2x + y = -3$
  $6x + 3y = -9$

48. $2x - y = 6$
  $-x + 8y = 12$

49. RENTALS To rent an inflatable trampoline for parties, it costs $75 an hour plus a set-up/tear-down fee of $200. Write an equation that represents this situation in slope-intercept form. (Lesson 2-4)

▶ GET READY for the Next Lesson

PREREQUISITE SKILL Find each value if $f(x) = 4x + 3$ and $g(x) = 5x - 7$. (Lesson 2-1)

50. $f(-2)$ **51.** $g(-1)$ **52.** $g(3)$ **53.** $g(-0.25)$